Is Covid-19 a Parallel Shock to the Term Structure of Mortality?

With Applications to Annuity Valuation

Moshe A. Milevsky[1]

[1]Professor of Finance
Schulich School of Business &
Graduate Faculty of Mathematics and Statistics
York University, Toronto, Ontario

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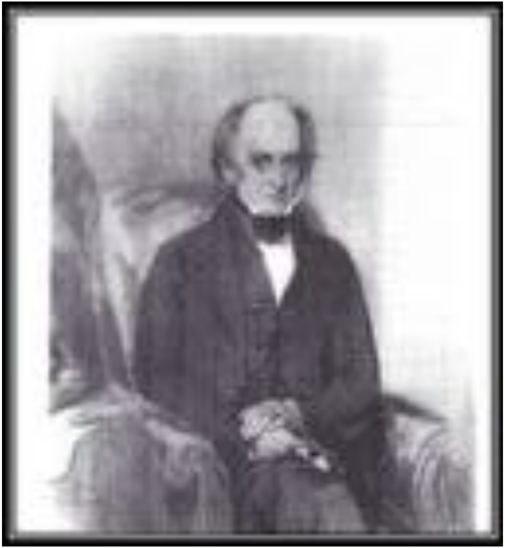
Lecture In Memoriam: Benjamin Gompertz



(born 1779, died 1865)

Context

These slides are from the One World Actuarial Research Seminar (OWARS), delivered on 20 May 2020, and was intended to provide an overview of my **ongoing work** with co-authors (T.S. Salisbury, H. Huang) and doctoral investigators (A. Nigiri, B. Ashraf). Over time we will complete and release associated working papers, so please view these results as preliminary.

Outline

This presentation focuses on how to think about and model covid-19 in continuous-time, from the perspective of life-cycle financial economics and retirement income planning. To begin with it hypothesizes that total mortality rates during the coronavirus period have been strictly proportional to normal mortality rates, which effectively increase biological ages across the curve, otherwise known as a parallel shift of the (Gompertzian) term structure. The presentation then goes on to provide some preliminary empirical evidence from the UK and Europe corroborating the parallel shift hypothesis, and discusses the implications of a (arguably, convenient) parallel shift on the utility-based valuation of life annuities. The main practical message here is that longevity insurance becomes **more valuable**, even if life expectancies decline. The presentation concludes by proposing the so-called compensation law of mortality (CLM) as a possible alternative to a parallel shift, and briefly discusses how to merge a CLM into a stochastic lifecycle model of investment and consumption.

Economic Motivation

1. The classical lifecycle model of financial economics suggests people (i.) borrow, (ii.) invest and (iii.) save, with an objective to **smooth consumption** over their life, adjusting for survival probabilities.

2. What is the impact of a sudden **shock to mortality**, such as the one generated by covid-19, within the context of the life-cycle model?

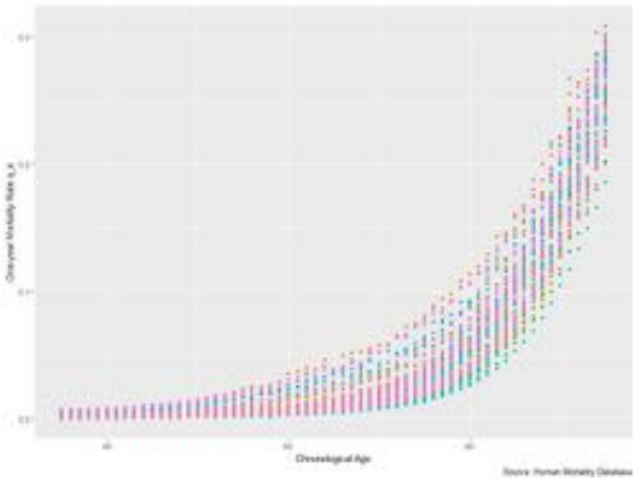
Outline of Lecture (50 minutes)

A glimpse of interests and ongoing work with co-authors, versus a coherent paper/thesis

1. What exactly do I mean by the so-called Term Structure of Mortality? What is a parallel shock (and what isn't)? Link to Biological Age.
2. Answer empirical (statistical) question: Is covid-19 a parallel shock?
3. Implications for annuity economics & retirement income planning.

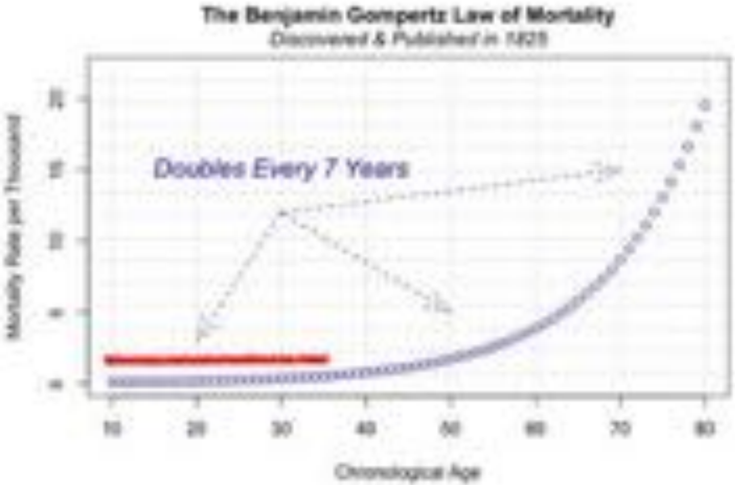
(Normal) Mortality Rates from Around the World

To begin with, there is quite a bit of variation around the world...



Actual q_x values from 37 countries in the year 2011, Source: HMD.

There is a Law Governing Death: (Over Adult Ages)



Grows by 10% per year.

Or: $q_{x+7} \approx 2q_x$

Warning: This Doesn't Work at Younger Ages

Remember the Distinction between Cohort vs. Period.

Visualizing two dimensions: (Thanks to Wilhelm Lexis)

	DEATH ->	2018	2019	2020	2021	2022
BORN						
1955		q.63	q.64	q.65	q.66	q.67
1954		q.64	q.65	q.66	q.67	q.68
1953		q.65	q.66	q.67	q.68	q.69
1952		q.66	q.67	q.68	q.69	q.70
1951		q.67	q.68	q.69	q.70	q.71
1950		q.68	q.69	q.70	q.71	q.72

Note: For the most part I'll be talking about the yellow (period) column, because it will be a while before we get data for the (cohort) rows.

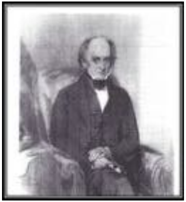
The Gompertz (1825) Law of Mortality

Like gravity, a law that's reasonably accurate even after two centuries...

The *natural* mortality rate at (chronological) age x is expressed as:

$$\mu_x \;-\lambda = \;\frac{1}{b}\,e^{(x-m)/b} \;=\; he^{gx}, \tag{1}$$

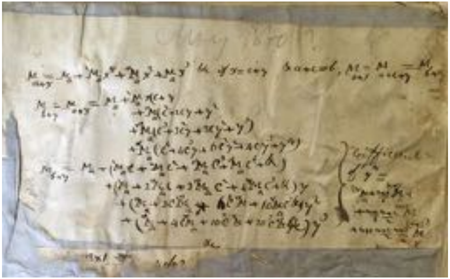
where m is a *modal* coefficient, b is a *dispersion* coefficient and λ is an accidental death rate. The (m, b) formulation is used in actuarial finance, but demographers and biologists tend to use (h, g) notation.

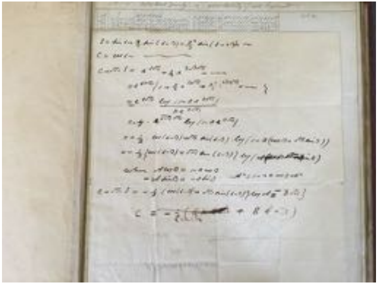


Note: Gompertz himself never really used either of these two expressions.

On a Historical Note: Gompertz & his Notes

From the Institute and Faculty of Actuaries, Staple Inn Actuarial Society

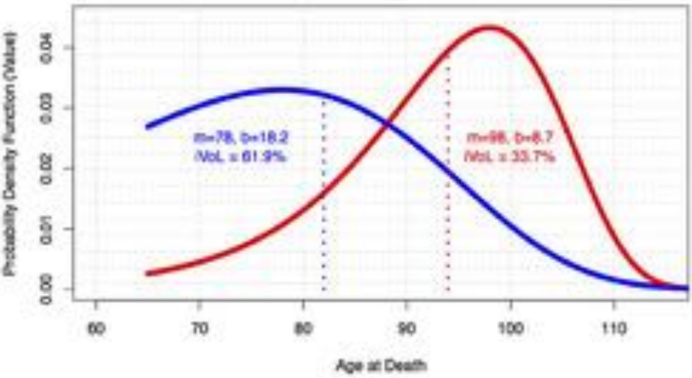




With permission and thank you to David Raymont, Librarian at the Institute and Faculty of Actuaries.

Visualizing the Remaining Lifetime: T_x

Intuition for the $m = \ln[g/h]/g$ and $b = 1/g$ parameters in the Gompertz Law



The coefficient of variation, or iVoL := $SD[T_{65}]/E[T_{65}]$, which I will label **individual longevity risk.**

The Term Structure of Mortality: Defined

Using the (demographic, biological) notation for the Gompertz law of mortality, I define the *natural* term structure of mortality (TSM) as:

$$\ln[\mu_x - \lambda] \; = \; \ln[h] + gx, \quad x >> 0, \tag{2}$$

where the accidental (Makeham) constant $\lambda << \mu_x$. Now, remember that the (log) survival probability is:

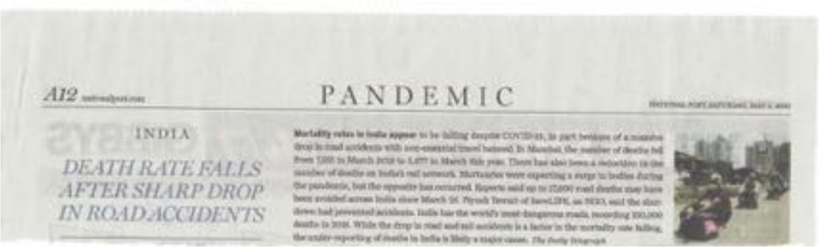
$$\ln[{}_t p_x] \; = \; -\int_x^{x+t} \mu_s ds \; = -\lambda t + \frac{he^{gx}}{g}\left(1 - e^{gt}\right). \tag{3}$$

So, the TSM is defined differently than in finance, where $\ln[{}_t p_x]$ would be divided by t, where: ${}_t p_x$ is the price of a zero-coupon bond maturing at t.

Accounting for Makeham (λ) is Important at All Ages

Here is yet another interesting byproduct of covid-19 and lockdowns

Up to 17,000 road deaths may have been avoided across India since March.



A12

PANDEMIC

INDIA

DEATH RATE FALLS AFTER SHARP DROP IN ROAD ACCIDENTS

Quick Technical Note: q_x versus μ_x

If $\mu_x = he^{gx}$, (i.e. $\lambda = 0$), the one-year death rate will be:

$$q_x \; := \; 1 - e^{he^{gx}(1-e^{g})/g}, \tag{4}$$

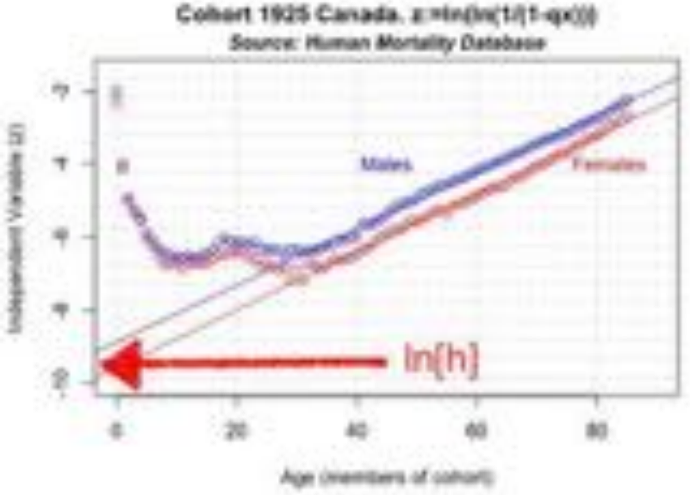
So, while μ_x increases exponentially, the one-year death rate does not. Generally speaking the data (e.g. HMD) is given as q_x (or m_x), so it's common to see the Gompertz assumption **approximated** as:

$$q_{x+t} \; \approx \; q_x e^{gt}, \tag{5}$$

which isn't the same as equation (4), and also ignores the Makeham term. To be precise, $z := \ln[\ln[1/(1-q_x)]]$ is a linear function of x, not $\ln[q_x]$.

The (1925 Cohort) Term Structure of Mortality

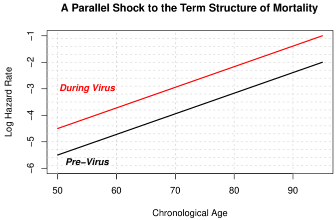
Notice the region in which the Gompertz model is reasonable.



Let's Talk Mortality Shocks (a.k.a. the virus)

Perspective #1

A parallel shock to the term structure of mortality is defined as: $\ln[\mu_x - \lambda]$ **increasing** by a **constant** v for all x in the Gompertzian age range.



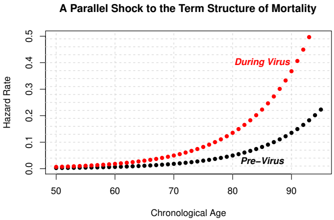
- Pre-Virus: $\ln[\mu_x - \lambda] = \ln[h] + gx$
- Virus: $\ln[\hat{\mu}_x - \hat{\lambda}] = v + \ln[h] + gx$
- Note: Assuming no change in g is problematic, and I'll return to this later.

What happens after the virus period?
Perhaps: $\ln[\mu_x - \lambda] = (\ln[h] - \kappa) + gx$.
See recent paper by A. Cairns and D. Blake:
http://www.pensions-institute.org/

Let's Talk Mortality Shocks (a.k.a. the virus)

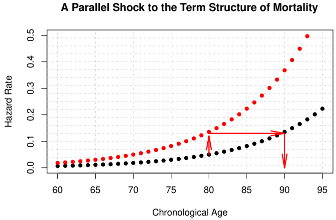
Proportional: Perspective #2

A parallel shock is defined as the *natural* hazard rate: $(\mu_x - \lambda)$ being **multiplied** by a **constant** e^v, during the period of the virus.



- Pre-Virus: $\mu_x - \lambda = he^{gx}$
- Virus: $(\hat{\mu}_x - \hat{\lambda}) = he^{(gx+v)}$
- Approximation in discrete time is: $\hat{q}_x \approx (1 + C)q_x$, for $C > 0$.
- The $C = \frac{\hat{q}_x - q_x}{q_x}$ is excess mortality.

A parallel shock to the term structure of mortality (TSM) is defined as **biologically aging** by v/g **years** in a Gompertzian framework.



- Pre-Virus: $\mu_x - \lambda = he^{gx}$
- During Virus: $\hat{\mu}_x - \hat{\lambda} = he^{g(x+v/g)}$
- For example, if $v = 1$, and $g = 10\%$, the virus *ages everyone* by 10 years.

If **biological age** was elevated to begin with (a.k.a. frail, co-morbidities) the impact is even greater!

What exactly is meant by Biological Age?

I'm **not** talking about clinical or molecular *biomarkers* of aging, such as epigenetic (CpG) clocks, DNA methylation, telomere length, etc.


REVIEW

Aging Cell WILEY

Measuring biological aging in humans: A quest

Luigi Ferrucci[1] | Marta Gonzalez-Freire[1] | Elisa Fabbri[1,2] | Eleanor Simonsick[1] | Toshiko Tanaka[1] | Zenobia Moore[1] | Shabnam Salimi[3] | Felipe Sierra[4] | Rafael de Cabo[1]

Abstract

The global population of individuals over the age of 65 is growing at an unprecedented rate and is expected to reach 1.6 billion by 2050. Most older individuals are affected by multiple chronic diseases, leading to complex drug treatments and increased risk

I mean a Mortality and/or Longevity Risk-Adjusted Age

Show me a mortality rate (curve) and I'll give you an age (function).

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

Calibrating Gompertz in reverse: What is your longevity-risk-adjusted global age?

Moshe A. Milevsky

See references (and distinctions) in that article.

Extreme Intuition for a Parallel Shock:

Impact on period life expectancy, survival probabilities and annuity prices, if permanent.

Assuming $ln[h] = -11.1$ (intercept), $g = 10\%$ (slope), which is based on Canada, with an $r = 3\%$ rate (and no loading.)

Shock	**Excess**	$E[T_{65}]$	**Bio.**	$({}_{30}p_{65})$	$a_{65}(r = 3\%)$
Baseline	**Mortality**	**Years**	**Age**	**Probability**	**Dollars**
Zero	0%	20.105	65	14.7%	\$14.398
$v = 0.20$	22%	18.534	67	9.6%	\$13.547
$v = 0.40$	49%	17.017	69	5.7%	\$12.690
$v = 0.60$	82%	15.560	71	3.0%	\$11.832
$v = 0.60$	The annuity factor if $r = 0.80\% \longrightarrow$				**\$14.398**

The mortality shock (making annuities cheaper)
can be wiped out by a decline in interest rates!

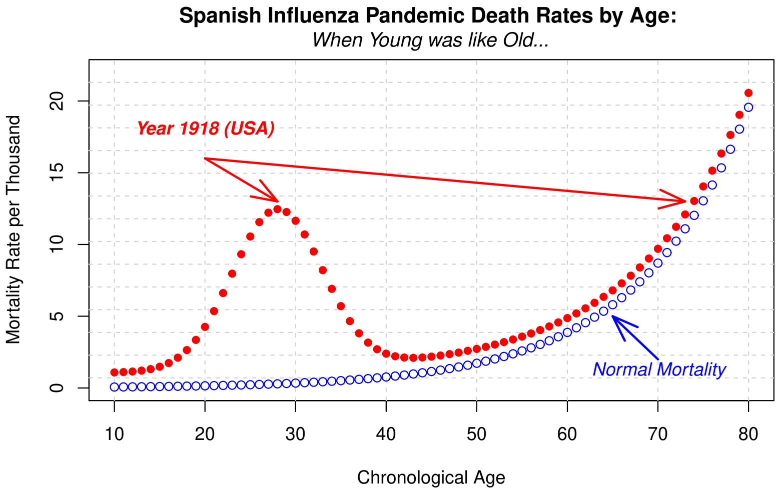
To Be Very Clear

For these (massive) declines in period life expectancy and/or increases in Biological Ages to take place, the shock to mortality (a.k.a. parallel shift in the term structure) would have to be permanent. Of course, nobody in their right mind believes (in May 2020) that we will continue to see these excess mortality rates for ever.

Rather, the point here is to help translate *excess mortality rates* into units that are more intuitive, namely life expectancy. Obviously, one can add a parameter to the period life expectancy calculation, perhaps the rate at which the mortality shock **decays**, and compute a more-realistic: $E[T_x]$.

A Non-Parallel Shock to the Term Structure of Mortality

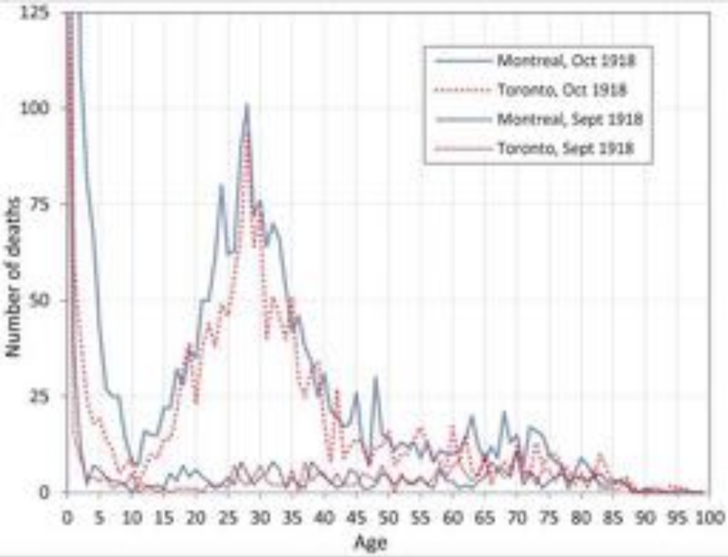
A Century-old Counter Example



Raw Data Source: Gagnon, et al. *PLOS One* (2013).

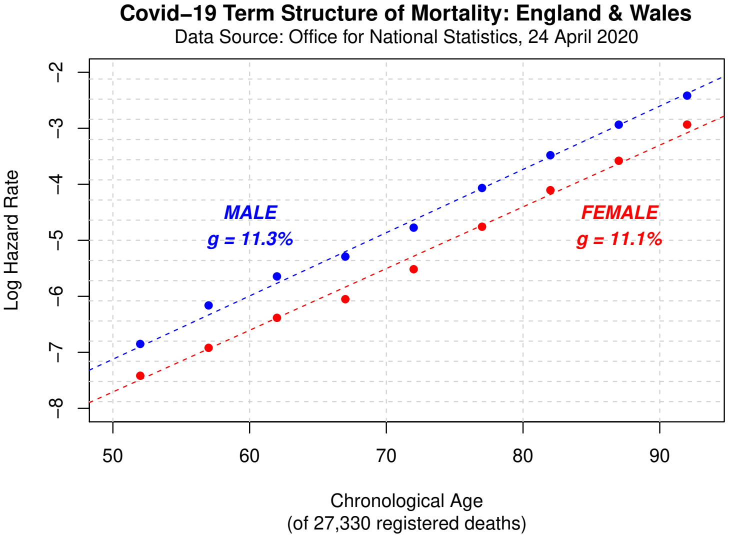
Age-Specific Mortality During 1918 Influenza Pandemic:

The raw data in Gagnon (2013), doi: 10.1371/journal.pone.0069586



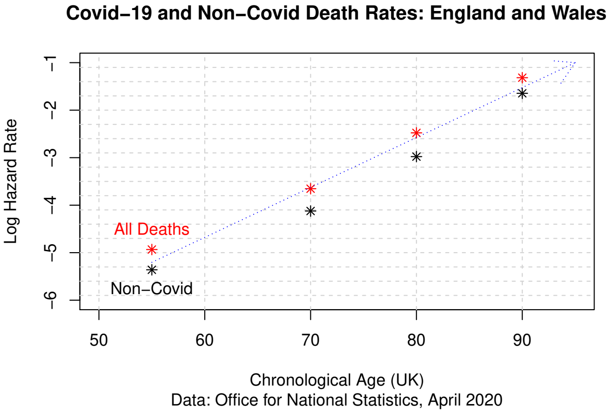
Is the Virus Gompertzian?

Preliminary evidence that Covid-19 is a parallel shock



Yes, but is it a parallel shock?

Perhaps the virus is correlated with non-virus mortality in a non-linear way?

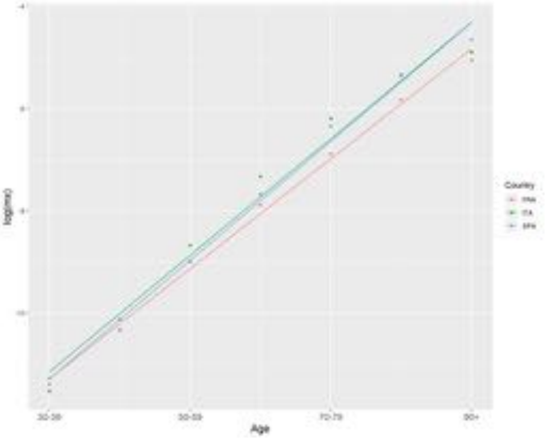


Warning: Work (and Data Collection) in Progress.

See work/blog by: David Spiegelhalter.

What About Other Countries?

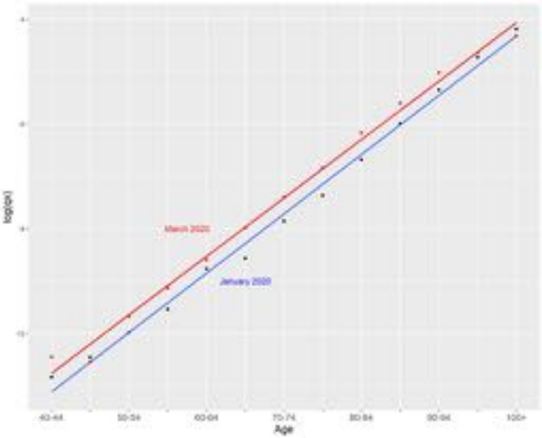
Project in progress with Andrea Nigiri, at Sapienza University of Rome.



Note: The slope and intercept are from a `glm`, with $g \in (0.107, 0.116)$.

Is it a Parallel Shock to the TSM?

Let's look at Deaths in Lombardia, Italy.

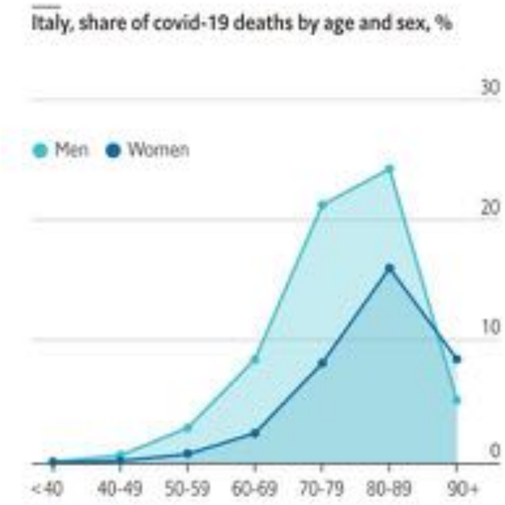


Blue line. All-cause mortality in January 2020 (pre covid)

Red line. All-cause mortality in March 2020 (during)

Parallel Shock!

So why do we see this picture so often?

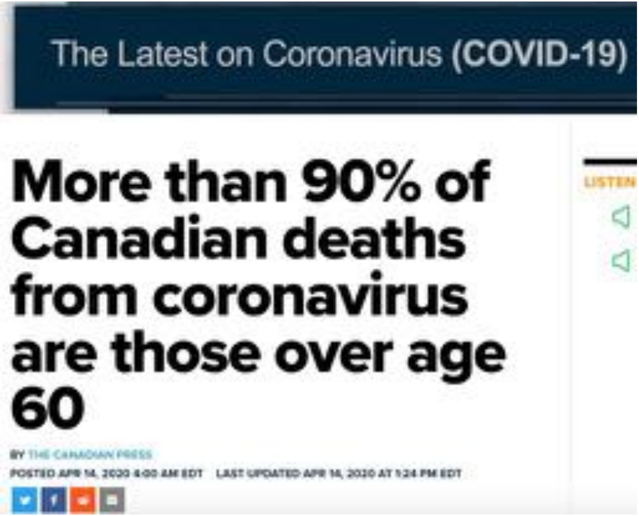


Parallel Shock?!

So why do we see this picture so often?

What fraction of (virus) deaths should be older?

Assuming a (virus that is a) parallel shock to the term structure of mortality



The Latest on Coronavirus (COVID-19)

More than 90% of Canadian deaths from coronavirus are those over age 60

BY THE CANADIAN PRESS

POSTED APR 14, 2020 4:00 AM EDT LAST UPDATED APR 14, 2020 AT 1:24 PM EDT

Distribution of Deaths vs. Death Rates

If the virus is a parallel shock to the term structure of mortality (TSM), and assuming an initial distribution (pyramid) for the population, what is the age distribution of deaths due to the virus?

Let's do some simple discrete mathematics

Ongoing work with T.S. Salisbury

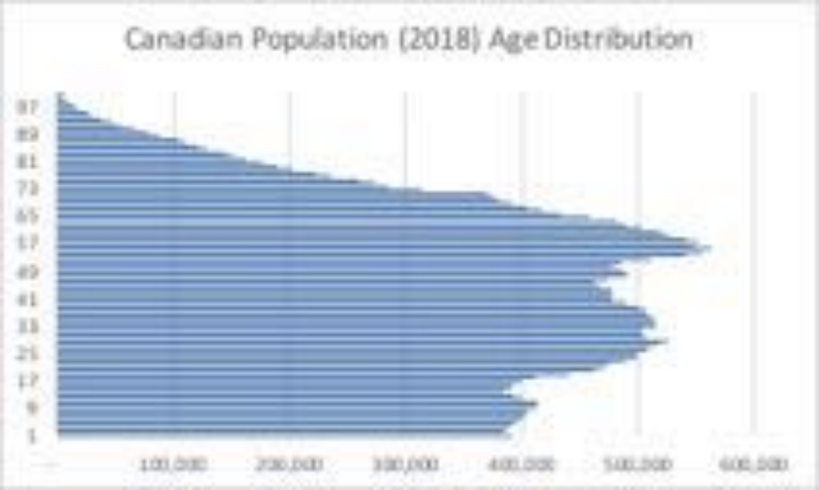
- Start with a discrete population (psi) distribution $\psi(x)$, $x = 0..120$, where x is age last birthday, and $N = \sum_i \psi(i)$ is the entire population (e.g. 37 million in Canada.)
- If the (non-shocked, normal) one-year death rate is q_x, we expect $\sum_i \psi(i) q_i$ deaths over the next year.
- A parallel shock to the term structure of mortality implies $\hat{q}_x \approx (1 + C) q_x$, where $C \geq 0$ is constant.
- The **fraction** of **excess deaths** over age x is the function (zeta):

$$\zeta(x) = \frac{\sum_{i=x}^{\omega} \psi(i)(\hat{q}_i - q_i)}{\sum_{i=0}^{\omega} \psi(i)(\hat{q}_i - q_i)} = \frac{\sum_{i=x}^{\omega} \psi(i) C q_i}{\sum_{i=0}^{\omega} \psi(i) C q_i} = \frac{\sum_{i=x}^{\omega} \psi(i) q_i}{\sum_{i=0}^{\omega} \psi(i) q_i}$$

- So, the size of C is irrelevant, and $\zeta(x)$ isn't affected by the virus, under a parallel shock.

Let's apply this to the population distribution in Canada

37 Million Canadians in the Year 2018: The $N\psi(x)$ function.



Numerical examples of the $\zeta(x)$ function

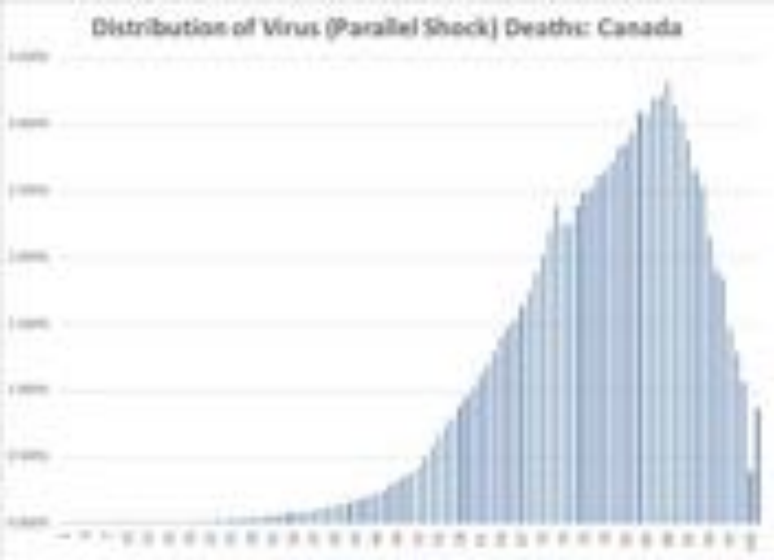
Within the Canadian population, what fraction of (excess) deaths will be...

Age Range	**Fraction of all Virus Deaths**
Under 50	$1 - \zeta(50) = 3.99\%$
50 to 59	$\zeta(50) - \zeta(60) = 7.64\%$
60 to 69	$\zeta(60) - \zeta(70) = 16.37\%$
70 to 79	$\zeta(70) - \zeta(80) = 25.09\%$
80 to 89	$\zeta(80) - \zeta(90) = 30.75\%$
90 & over	$\zeta(90) = 16.17\%$

Again, this does **not** depend on the size of the mortality shock, and we **should** expect to see most (88.4%) of the (excess) deaths above age 60.

A Visualization of the $\zeta(x)$ function

Based on the Population Distribution of 37 Million Canadians (in 2018)



Who Cares for the Benefits of a Parallel Shock?

We get closed-form expressions for actuarial & economic quantities of interest

If the TSM is shocked by adding a constant v to $\ln[h]$, then I can easily analyze a number of financial & economic expressions of interest.

$$a_x \;=\; \frac{\Gamma(-r/g, he^{gx}/g)}{g\exp\{(-1/g)(he^{gx} + r\ln[he^{gx}/g])\}}, \tag{6}$$

where $\Gamma(.,.)$ is the incomplete Gamma function. Derivation in appendix of Milevsky (JPEF, 2020), originally suggested by John Mereu (TSA, 1962).

Annuity equivalent wealth, per Kotlikoff and Spivak (JPE, 1981), or Brown (JPubE, 2001) a.k.a. subjective value from annuitization per \$1, is:

$$1+\delta \;=\; \left(\frac{a_x}{a_{\tilde{x}}}\right)^{\gamma/(1-\gamma)} \tag{7}$$

where γ is longevity risk aversion, and $\tilde{x} = x - \ln[\gamma]/g$. Source is Milevsky and Huang (NAAJ, 2018), or Cannon and Tonks (2008), equation (7.57).

The Utility Value of Annuitization (after a Virus Shock)

Longevity insurance is more valuable *even* though survival rates have declined.

Virus reduces life expectancy, but utility value of annuitization increases because **longevity risk** (a.k.a. the coefficient of variation) has gone up.

Shock Baseline	$E[T_{65}]$ Years	$SD[T_{65}]$ Years	**Coefficient of Variation**	\$1 **Annuity Equivalent**
Normal Times	20.11	9.11	45.3%	\$1.535
$v = 0.20$	18.53	8.74	47.2%	\$1.585
$v = 0.40$	17.02	8.36	49.1%	\$1.639
$v = 0.60$	15.56	7.95	51.1%	\$1.700

Assumes: $ln[h] = -11.1$, $g = 10\%$ (mortality growth rate), calibrated to Canadian HMD values, under an $r = \rho = 3\%$ interest rate. Formula in Milevsky & Huang (2018), based on the *Annuity Equivalent Wealth* in Brown (2001), originally defined by Kotlikoff & Spivak (1981), with $\gamma = 4$.

Yes, a Parallel Shock is a Straight Jacket

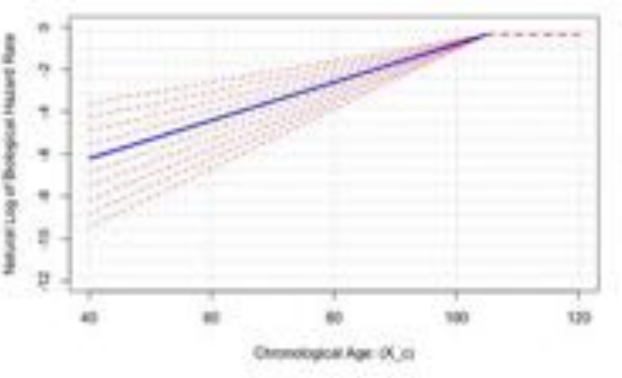
Statistically it can't be a *perfect* parallel shock.

1. A parallel shift implies excess mortality: $C = \frac{\hat{q}_x - q_x}{q_x}$ is *approximately* equal across all (Gompertzian) ages x.
2. But, there will always be noise in the data!
3. What if we observe the ratio $\hat{q}_x / q_x$ declines at very advanced ages?
4. What if fewer centenarians are dying from the virus than expected?
5. That might be evidence of a *compensation* law of mortality at work.

Compensation Law of Mortality (CLM)

Nature doesn't allow parallel shifts to the TSM

The Extremely Strong Version of a Compensation Law



Source: Gavrilov and Gavrilova (1991), *The Biology of Lifespan*

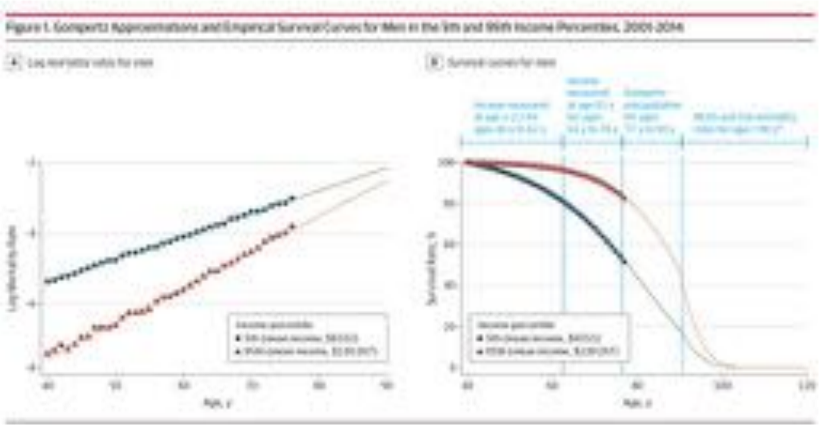
Clarification

To be very clear, I am **not** suggesting that in reality the various term structures of mortality (TSM) for *heterogenous* groups converge to a fixed and rigid plateau at some advanced age x^*, nor did Gavrilov and Gavrilova (1991) suggest this. Likewise, I'm not making any statements about whether $x^* = 100$ or perhaps as high as $x^* = 110$. Moreover, while nature might want to compensate mortality, she surely hates non-differentiable curves! Rather, I'm suggesting that this is reduced-form model is a helpful way to think about what is happening. I really don't need thousands of different mortality tables and hundreds of different improvement factors to get to the *financial and economic essence* of the matter.

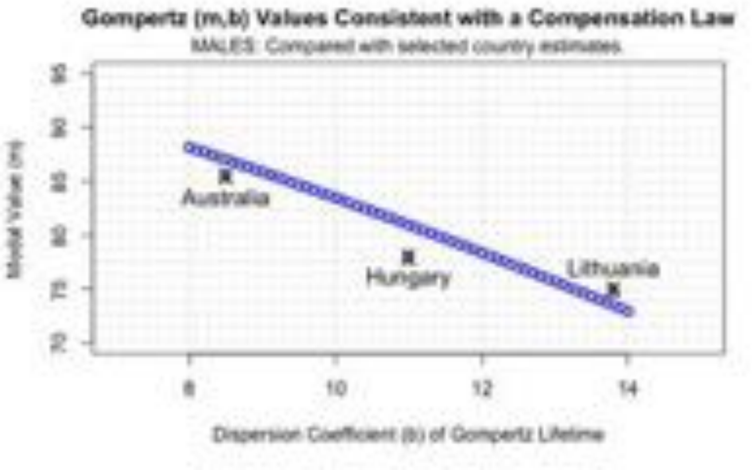
On a related note, see the recently published work by S.J. Richards (SAJ, 2020) on the minimal number of factors needed to parameterize mortality.

Empirical Support for Mortality Compensation

Chetty R, Stepner M, Abraham S, et al., JAMA. 2016, 315(16):1750-1766.



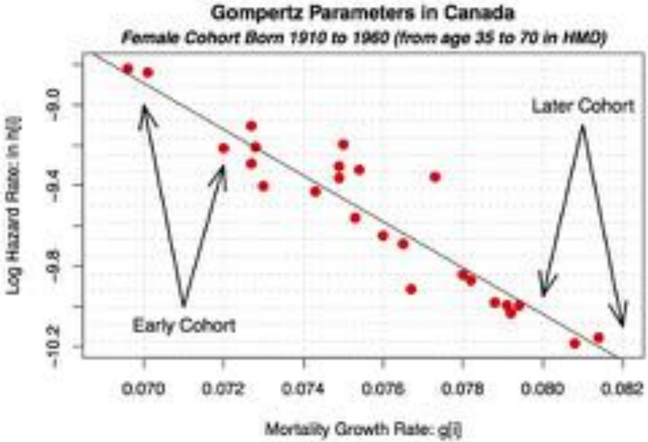
The higher your mortality rate (e.g. poor vs. rich), the lower your mortality growth rate. Think of $\ln[h]$ versus g.



Source: Milevsky (IME, 2020)

Mortality Compensation Effects by Cohort Group

Historical Evidence from Canada



A parallel (negative) shock (a.k.a. mortality improvement) would imply a vertical drop in the red dots over the years, without moving right.

So, nature forces a reduction in g when increasing $\ln[h]$.

This is the difference between a *parallel* shock versus a *constant* shock

What will the excess deaths fraction $\zeta(x)$ look like, when the shift from $\ln[h]$ to $\ln[h] + v$, is non-parallel? In other words, the intercept increases by v but the slope g declines to *compensate* for the increased mortality?

By definition of this *extreme compensation*, (log) hazard rates must all be equal at some age x^*, so the only way for this to happen is if a shock v is associated with a decline in the mortality growth rate from g to $g_v := (g - v/x^*) < g$. Technically:

$$\ln[h] + gx^* = (\ln[h] + v) + \overbrace{(g - \frac{v}{x^*})}^{g_v} x^*$$

So, if you want to shock the curve by $v = 0.40$ units, and $x^* = 100$, then you must reduce the mortality growth rate from $g = 10\%$ to 9.6%.

If a shock to $ln[h]$ is associated with a reduction in the mortality growth rate g, then the utility value of annuitization is even higher, because individual longevity risk which is proportional to $1/g$, is greater.

Finally: Modeling this in a **cohort** lifecycle framework.

Ongoing work with T.S. Salisbury, H. Huang (& B. Ashraf).

Let $h_t = \ln[\mu_t - \lambda]$ denote the **individual** log hazard rate **before** the virus, and assume it obeys the stochastic process:

$$dh_t \;=\; \xi \left(\frac{k - h_t}{T - t} \right) dt \,+\, \sigma dB_t, \;\; 0 \leq t < T, \tag{8}$$

where B_t is a BM, the *pinning* constant $k := \ln[\mu_{x+T} - \lambda]$, which is the log hazard rate at plateau time T, or age $(x + T)$. The parameters (ξ, σ) are the reversion speed and volatility. Importantly, $h_0 = \ln[\mu_x](1 + f)$, is the population (current age x) log hazard rate, and f is a measure of frailty, per the (classic) work of Vaupel, Manton and Stallard (1979).

And, anyone who manages to reach age $(x + T) \approx 100$ continues to *live* with a constant hazard rate, under an exponential lifetime distribution. This is the *world* before the virus shock.

What Does a (Constant Shock) Virus Do to the TSM?

Assume that a virus shifts the initial log hazard rate by a constant v, **and** in addition it changes the reversion speed to $\hat{\xi}$ and volatility to $\hat{\sigma}$. So, the stochastic differential equation for (what I am now calling) $\hat{h}$, is:

$$d\hat{h}_t \; = \; \hat{\xi}\left(\frac{k - \hat{h}_t}{T - t}\right) dt \, + \, \hat{\sigma} dB_t, \;\; 0 \leq t < T, \tag{9}$$

where (to be very clear), the pinning value remains k, and the Brownian motion driving the process remains B_t.

Financial Economic Question: How does the shock affect lifecycle consumption?

Final Thoughts (and Slide)

My email: milevsky@yorku.ca

1. Covid-19 is aging us all.
2. The odds of becoming a centenarian might have gone down slightly, due to the virus, but individual longevity risk has actually increased.
3. The utility value of annuitization has increased, and even more so if you believe a *mortality compensation* effect is at work.
4. Covid-19 *appears* Gompertzian and total mortality rates *appear* consistent with a parallel shock to the term structure of mortality.
5. Finally, hopefully this episode has convinced (classical) life-cycle economists that there is a need for stochastic mortality models.

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