

*Advanced Personal Finance: Retirement Income Models*

## Course Outline

Winter 2018

beginning on Jan 4<sup>th</sup>, 2018

Thursday, 11:30am to 2:30pm, Room S128

### Instructor

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### Assistant

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### Brief Description

This course addresses advanced topics in personal finance. It focuses on the management of uncertainty (randomness) consumers face towards the end of the lifecycle in the areas of longevity, mortality, inflation, investment returns, pensions and income taxes. The instructional pedagogy is interactive and computational. Students will learn how to create R-scripts that optimize and solve real-world retirement income problems.

Prerequisites/Corequisites/Course Exclusions:

Prerequisite: FINE 2000 3.0

Corequisite: FINE 4050 3.0

### Deliverables at a Glance

In the table below, the impact of each task on your final grade for the course is indicated in the “% weight” column.

Assignment/Task	Quantity	% Weight	Total %	Author
Homework Assignments1	1	20%	20%	Individual
Homework Assignment 2	1	30%	30%	Individual
Technical Group Project	1	40%	40%	Group
Class participation	1	10%	<u>10%</u>	Individual
			<b>100%</b>	

For details, see “Written Assignments/Projects and Exam[s]: Descriptions” (p.7 and “Evaluation ...” (p. 7)

## Course Material

*Required reading* for this course includes the following book[s]. It is (They are) available for purchase from the York University Bookstore (<http://bookstore.blog.yorku.ca>) or free online:

1. Moshe A. Milevsky, *Retirement Income Recipes in R*. (Note: This book is a work in progress and scheduled to be completed during 2018. All students will get a free copy of the chapter modules and material as they are being created and required week by week.) **[RiRiR]**
2. Moshe A. Milevsky, *The Calculus of Retirement Income: Financial Models for Pension Annuities and Life Insurance*, Cambridge University Press, March 2006 **[CRI]**. All students registered in this course (after the 3<sup>rd</sup> week) will get a complimentary copy of this book from the instructor. There is no need to purchase this book.
3. N. Charupat, M.A. Milevsky and H. Huang, *Strategic Financial Planning over a Lifecycle: A Conceptual Approach to Personal Risk Management*, Cambridge University Press, June 2012 **[SFP]**. Students should already have this textbook from their FINE4050: Personal Finance course which is a co/pre requisite. This will be used in the early weeks of the course.
4. Moshe A. Milevsky, *Life Annuities: An Optimal Product for Retirement*, CFA Institute, June 2013. Charlottesville, Virginia (this monograph is available for free as a PDF download from the CFA website.) **[LiA]**

There will be no additional reading material or textbooks assigned other than the material listed above. The bulk (70% to 80%) of the course content is contained within **[RiRiR]**, and the other books are included for reference and/or background purposes only.

Note that a few complimentary copies of **[CRI]** will be placed on special reserve (reference) in the Bronfman library immediately and complimentary copies will be available a few weeks into the course. So, assuming students have copies of **[SFP]** from the FINE4050 pre-requisite, there will be no need to purchase or acquire any additional textbook or course material. All lecture (module) notes and *R-scripts* will be placed on the CMD after the lecture.

### Software & Computer Requirements

Students will be required to download and install a copy of *R-studio* (open source, free) on their laptop computer. This program is widely available and extensively used among 'quants' working in financial services. I do not anticipate any problems (it is user friendly) and it will not require any assistance or support or (new) resources from Schulich's Computing Services group. Students will **not** require any prior experience or knowledge working with *R-scripts*. Note that R has excellent data-manipulation, illustration and graphing capabilities, which makes it preferable to Excel for working with large datasets and complex algorithms.

## Class-by-Class Syllabus

Generally speaking, the first 75 minutes of class will be a lecture by the instructor in which the context and the specific “problem” to be solved will be presented. After the break, the second part will consist of solving (hands-on) exercises using (the software program) **R**. Each of the 12 lectures corresponds to one module in the (to be completed) textbook [**RiRiR**] and will be made available to the students on the CMD.

Topics, readings, and other preparations for every class are listed below

Note: If any changes in this schedule become necessary, notifications will be posted on the course CMD, and when changes need to be announced between classes, an email will be sent to students’ Lotus Notes email accounts, notifying them of the change.

<p><b>Session #1</b> <b>Thu/Jan/4</b></p>	<p><b>Background &amp; Social Context.</b> Statement of the main challenge. Rational economic theory dictates that a consumer with no labor income (zero human capital value) will want to “smooth” consumption over the remainder of their lifetime, perhaps with some funds set-aside for legacy. If the elderly knew (i.) exactly how long they were going to live, (ii.) the real interest rates and investment returns they will earn, (iii.) the income tax rates they will experience, and (iv.) the medical costs they will incur, retirement income planning would be an easy problem to solve. Questions addressed: Assuming that you lived on a distant planet with absolutely no uncertainty or (income taxes) what fraction of your salary should you save at the age of 40, 50 and 60 if you want to maintain and enjoy a relatively constant standard of living over the course of your entire life? What multiple of your salary should be accumulated or stockpiled in your retirement account (nest egg) at the age of 40, 50 and 60 in order to maintain the optimal trajectory? How does the answer depend on whether you plan to retire at age 70 and live to age 90 vs. retire at the age of 80 and live to 100? Of course, I plan to live for a very long time, but the point is to see how the horizon affects (or maybe not) the results. <b>WHAT ARE THE PROBLEMS WE WILL SOLVE IN THIS COURSE?</b></p> <ol style="list-style-type: none"> <li>a. <u>Technical Skills Acquired:</u> Students review the lifecycle model (<b>LCM</b>).</li> <li>b. <u>Reading Material:</u> [<b>RiRiR</b>] Module and [<b>SFP</b>] chapter #2 and #3.</li> <li>c. <u>Learning Outcome:</u> Students will learn the main challenges facing (elderly) retirees in the 21<sup>st</sup> century as well as the business opportunities.</li> </ol>
<p><b>Session #2</b> <b>Thu/Jan/11</b></p>	<p><b>A Self-contained Crash Course on What You Need to Know about R.</b></p>
<p><b>Session #3</b> <b>Thu/Jan/18</b></p> <p><b>HW #1 Due at the end of class.</b></p>	<p><b>Longevity of Money.</b> Computing the “ruin time” of an investment portfolio under fixed investment returns and fixed inflation-adjusted withdrawals. A quick review of the time value of money (present value and future value) under a full term-structure (or yield curve) of interest rates. Questions addressed: You are 70 years-old with \$1,000,000 in an investment account earning 6% compound interest every year, but you are withdrawing and spending \$80,000 (i.e. 8% of your initial nest egg) every year. Ignoring income taxes for the moment, how long does the money last? What is the longevity of the portfolio? Now continue to assume that you are earning 6%, but that every year you are withdrawing 8% of the value of the account, in how many years will the account be worth less than \$1?</p>

	<p>What if you increase your yearly withdrawals by an inflation factor (for example 2%)? Now what is longevity of the portfolio?</p> <ol style="list-style-type: none"> <li><u>Technical Skills Acquired</u>: Continue to learn <i>R-scripts</i>.</li> <li><u>Reading Material</u>: [RiRiR] Module.</li> <li><u>Learning Objectives</u>: Understanding the concept of portfolio longevity.</li> </ol>
<p><b>Session #4</b> <b>Thu/Jan/25</b></p>	<p><b>Mechanics of Defined Benefit (SB) pensions.</b> Careful review of the Canada Pension Plan. A short digression to review the current rules regarding eligibility, structure and payout from CPP as well as OAS &amp; GIS claw-backs. Discuss internal rates of return (IRR) as well as subsidies and transfers. Questions addressed: You are a member of a DB pension plan that promises to provide you with a retirement income for the remainder of your life, based on a formula. You contribute 4% of your salary and your employer contributes 6% of your salary to this plan up to \$50,000 per year of your salary. The formula stipulates that for every year of work and contribution to the plan you are entitled to 2% of your salary (again, up to \$50,000) in the final year of work, but this is capped at 70% of your final salary. The rules can be complicated, but what is the internal rate of return (IRR) from this pension plan if you start contributing at the age of 25, retire at the age of 65 and live to age 95? To what age would you have to live to earn a 4% IRR from you pension? If you work from 20 to 70, then retire and die at 100, would you have been better off investing the money in a savings account earning 1% real (inflation adjusted) interest every year?</p> <ol style="list-style-type: none"> <li><u>Technical Skills Acquired</u>: Computing your CPP benefit in <i>R-script</i>.</li> <li><u>Reading Material</u>: [RiRiR] Module and current CPP rules (from website)</li> <li><u>Learning Objectives</u>: A detailed understanding of rules governing the Canadian Pension Plan (CPP) with some international comparison.</li> </ol>
<p><b>Session #5</b> <b>Thu/Feb/1</b></p>	<p><b>By this point in the course you should be (very) comfortable working with R. Initial Project Assessment to be discussed with groups. Details to follow.</b></p>
<p><b>Session #6</b> <b>Thu/Feb/8</b></p>	<p><b>Implementing non-linear income taxes.</b> Inverting the tax function and the tax treatment of different (retirement) investment accounts, such as RRSPs, TFSA, and fully taxable accounts. The Registered Retirement Income Fund (RRIF) rates and rules. Discussion of taxes due upon death and the single vs. joint problem. Questions addressed: Assume that you have equal amounts of money in three different types of tax-accounts. The first (A) is a fully taxable account in which you pay income taxes on all realized gains. The second (B) is a tax-deferred account. You received a tax deduction when you contributed (added) funds to this account, so when you (eventually) make withdrawals they will be taxed at your marginal tax rate. The third (C) account is a tax-free savings account in which all investment gains aren't taxable and you can withdrawal as much as you want without paying any income taxes. Now assume that you would like to live on a fixed after-tax income every year during retirement. What is the optimal sequence of withdrawals to maximize the longevity (as defined in module #2) of your money? Which accounts do you empty first/last if you want so smooth your retirement income (as defined in module #1)?</p>

	<p>How exactly does it depend on your marginal tax rate and/or the type of investments you are holding in your taxable account?</p> <ol style="list-style-type: none"> <li><u>Technical Skills Acquired</u>: Coding Federal and Provincial tax schedules in <i>R-script</i> and solving optimization problems.</li> <li><u>Reading Material</u>: [RiRiR] Module and Current Income Tax Regulations.</li> <li><u>Learning Objectives</u>: How do income taxes complicate the process of consumption smoothing?</li> </ol>
<p><b>Session #7</b> <b>Thu/Feb/15</b></p>	<p><b>Long-term Investment Returns and Interest Rates.</b> Discussion and review of historical investment returns from various asset classes to simulate and/or forecast portfolio behavior over the long run. Plausible models for equity and bond returns. Questions addressed: You have \$100,000 invested in a stock-index fund and you are adding or saving \$1,000 to this account every month. This particular fund has been in existence for 50 years (600 months). What is a reasonable estimate or range for what your investment account will be worth in 5, 10 or 25 years on a pre-tax basis? What will it look like on an after-tax basis and how does it depend on realized vs. unrealized investment gains? What if you diversify your portfolio and add a bond fund to the mix? More importantly, what will the investment account be able to purchase on an after-inflation basis? Given the limited history, how confident can we be with the answer?</p> <ol style="list-style-type: none"> <li><u>Technical Skills Acquired</u>: Simulating asset class returns using R-scripts.</li> <li><u>Reading Material</u>: [RiRiR] Module and historical returns database.</li> <li><u>Learning Objectives</u>: Intelligent conversation on long-term returns.</li> </ol>
<p><b>Thu/Feb/22</b></p>	<p><b>Reading week. No class.</b></p>
<p><b>Session #8</b> <b>Thu/Mar/1</b></p> <p><b>HW#2 Due</b></p>	<p><b>Simulating Withdrawal Strategies.</b> Investigating the Sequence of Investment Returns. How long does a portfolio last (in retirement) when investment returns are stochastic? How does the longevity of the portfolio correlate with the realized returns during various sub-periods? How can non-linear instruments (such as put and call options) be used to protect a portfolio in withdrawal mode? Questions addressed: Combine the motivating question from module #2 and module #5. Assume that you retire with \$500,000 in an account that is 100% invested in a diversified stock-index fund and you are withdrawing \$25,000 every year from this account, adjusted by inflation. The longevity of the portfolio is now random. What is a reasonable estimate or range for how long the money will last? How sensitive is the answer to the (random) investment performance of the account during the first few years? Could the portfolio longevity range be improved if you allocated part of the account to bonds or even to risk-less cash? Are there other ways -- for example using derivative securities such as put and call options -- or techniques to extend the longevity range and life of the retirement account?</p> <ol style="list-style-type: none"> <li><u>Technical Skills Acquired</u>: Monte Carlo simulation using R-script.</li> <li><u>Reading Material</u>: [RiRiR] Module</li> <li><u>Learning Objectives</u>: In depth understanding of the so-called Sequence of Returns (Sor) risk and the impact on accumulation vs. de-accumulation.</li> </ol>

<p><b>Session #9</b> <b>Thu/Mar/8</b></p>	<p><b>Modeling Random Lifetimes.</b> Understanding Mortality Tables and Longevity Projections. How long do people live? How random (uncertain) is lifetime? What does it depend on? How do statisticians (actuaries) model the remaining lifetime? How has this changed over the centuries? What are the statistical distributions used to analyze mortality and longevity? Questions addressed: The length of human life is (obviously) random and the odds that a 70-year old will survive or live for another 5 years are (much) higher than they are for a 90-year-old. But, what are the probabilities (exactly) and where do they come from? What exactly is a mortality table? Why are there so many of them and why are they outdated? How do I know which one (of the many) to select when reporting survival and mortality probabilities? How do I adjust these numbers for couples and joint-survival probabilities?</p> <ul style="list-style-type: none"> <li>a. <u>Technical Skills Acquired:</u> Download mortality tables from the Society of Actuaries (SoA) and Human Mortality Database (HMD) at Berkley. Basic computations using R-script.</li> <li>b. <u>Reading Material:</u> [RiRiR] Module.</li> <li>c. <u>Learning Objectives:</u> Understanding &amp; working with randomness in life.</li> </ul>
<p><b>Session #10</b> <b>Thu/Mar/15</b></p>	<p><b>Continuous Laws of Mortality.</b> Develop (convenient) approximations to discrete mortality tables. The force of mortality as a force of interest. Introduction to the Gompertz-Makeham model used for modeling mortality. Contrast with an exponential distribution assumption for future lifetimes. Can life be normal? Why does the Bell curve not work? Questions addressed: Ok, I know how to use a mortality table to compute the odds of dying or living in any given year, but how can I use that table to compute a 65-year-old's expected remaining lifetime? More importantly, since it is obviously random, what is the variance (or standard deviation) of their remaining lifetime? What is more uncertain? Is it your remaining lifetime? Or is it the investment return on your portfolio? How do these concepts relate to the (commonly heard and widely abused) term longevity risk? Are there any (easy) formulas that can be used to compute the mean and standard deviation or the moments of the distribution of remaining lifetime? What will these moments look like for a 65-year-old, but in 10, 20 or 30 years from now? Are there any patterns in the mortality data? Does death follow any rules? I have heard that 50% of babies born this year will live to the age of 100. What sort of improvement (reduction) in current mortality rates would be required to achieve these probabilities?</p> <ul style="list-style-type: none"> <li>a. <u>Technical Skills Acquired:</u> Fitting various survival curves using continuous approximations in R-script.</li> <li>b. <u>Reading Material:</u> [RiRiR] Module &amp; [CRI] chapter #5.</li> <li>c. <u>Learning Objectives:</u> Appreciate the biological and historical basis of various laws that govern human mortality. Implication for pricing.</li> </ul>

<p><b>Session #11</b> <b>Thu/Mar/22</b></p>	<p><b>Life Annuities and Life Insurance.</b> Pensions in Discrete and Continuous Time. Advanced-Life Delayed Annuities (ALDA), the tax-treatment of annuity income as well as registered and non-registered annuities in Canada. Questions addressed: I am 70 years-old and being offered a so-called (pension) annuity that will pay me 10\% income for the rest of my life, but the purchase is completely irreversible, the product is illiquid and if-and-when I die my heirs will receive absolutely nothing. The money is lost. Is this a fair deal? What mortality (table) assumptions underlie this payout rate? The insurance company is willing to refund the money to my heirs when I die, but that will reduce the payout from 10% (initial) to only 6%? Is that a better deal? Or should I take the higher 10% and use some of the money to purchase life insurance (with a payout to my heirs)? What metric of formula should I use to properly compare all these (pension) annuity options on both a pre-tax and after-tax basis? Why do the payout rates (and taxable portion) differ across companies?</p> <ul style="list-style-type: none"> <li>a. <u>Technical Skills Acquired</u>: Creating <i>R-scripts</i> to price all forms of annuities.</li> <li>b. <u>Reading Material</u>: [RiRiR] Module #9 &amp; Chapter #1, #2 of [LiA].</li> <li>c. <u>Learning Objectives</u>: What is the proper price for a pension annuity?</li> </ul>
<p><b>Session #12</b> <b>Thu/Mar/29</b></p>	<p><b>Mortality and Longevity Derivatives.</b> Pooling of risk. Pricing by equivalence. Reserves and capital requirements. Questions addressed: An insurance company is offering me a (variable annuity) product that appears to combine elements of a mutual fund and life annuity. It is called a Guaranteed Lifetime Withdrawal Benefit (GLWB) and this is how it works: A \$100,000 investment can be allocated to a combination of stocks and bonds which will grow (randomly) over time and is completely liquid. Once I reach the age of 65 I can withdraw 5% of the (then) account value for the rest of my life, even if the account is emptied and hits zero. But, if I wait until age 70 the company offers 6% of the account value. Can I manufacture this (derivative) by allocating some of my money to a cheaper mutual fund and part to a life annuity? Should I turn-on the income at age 65 (getting 5%) or wait until age 70 (for the 6%)? More importantly, is it worth paying 100 basis points every year for this option?</p> <ul style="list-style-type: none"> <li>a. <u>Technical Skills Acquired</u>: Using <i>R-scripts</i> to analyze GLWB payouts.</li> <li>b. <u>Reading Material</u>: [RinR] Module</li> <li>c. <u>Learning Objectives</u>: Can a GLWB with synthesized?</li> </ul>

## Written Assignments/Projects and Exam[s]: Descriptions

<i>Due Date</i>	
<b>Jan/18</b>	<p><u>Homework Assignment 1</u></p> <p>This is a technical assignment <b>due on Jan/18</b> that will help (force) student to familiarize themselves with basic computations in R. It will involve the use of <i>R-scripts</i> and various functions included in the standard package.</p> <p><i>Max length:</i> There is no page limit on this assignment. <i>Value:</i> 20%</p>
<b>Mar/1</b>	<p><u>Homework Assignment 2</u></p> <p>This is a technical assignment <b>due on Mar/1</b> that will require students to download and perform various calculations involving simulated investment returns.</p> <p><i>Max length:</i> There is no page limit on this assignment. <i>Value:</i> 30%</p>
1 week after last class	<p><u>Technical Group Project</u></p> <p>Students will be placed into groups of (no bigger than) three <b>by Week #6</b>. Although the precise topic of the project itself will be discussed and agreed with the instructor, the underlying objective (and deliverable) will be a <b>detailed analysis</b> of a Retirement Income strategy or product. Examples of such products or strategies include, Sustainable Withdrawal Rates, Guaranteed Living Withdrawal Benefits, Tontine Annuities, Mediaeval Corrodies, Income Buckets, Longevity Insurance, etc.</p> <p><i>Max length:</i> The deliverable is a 15-page description &amp; write-up that is due no later than ONE WEEK after the last day of class.</p> <p><i>Value:</i> 40%</p>
	Note: +10% participation